

OPTIMISING THE FREQUENCY AND RESOLUTION OF THE RPT TECHNIQUE FOR ANALYSING FLUID DYNAMICS IN SMR REACTORS

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ABSTRACT

Radioisotopes are widely used in research and technological development due to the precision of their measurements and the possibility of applying them in non-intrusive methods. One of these uses is the Radioactive Particle Tracking technique, which allows processes in dynamic systems to be monitored and analysed. This work investigates the relationship between the spatial resolution of the detection system and the frequency of data acquisition. This, in turn, is a critical factor, as an inadequate acquisition frequency can introduce noise into the data or cause errors in the signal reconstruction. The study considers a radioactive source moving along the X axis and a detection system with N detectors. The sensitivity and spatial resolution of the system were mathematically deduced by relating the resolution, the particle velocity, the data acquisition frequency and the geometry of the experimental setup. The study also describes how the geometric efficiency of the detectors influences the variation in counts, which is fundamental to determining the system's resolution. The results show that resolution improves with increasing acquisition frequency up to a certain point, but reaches a plateau where further improvements become insignificant or even detrimental due to noise. The 200 Hz frequency provided the best resolution, while 10 Hz resulted in the worst, which exposes the importance of selecting the optimal frequency to maximise accuracy without introducing noise. The conclusion reaffirms the effectiveness of the mathematical model developed, which can be used to optimise the experimental setup and the minimum required particle activity. Furthermore, the appropriate choice of acquisition frequency is essential to guarantee the best performance of the detection system, balancing the resolution and efficiency of the system. This work provides a robust theoretical basis for future experiments and practical implementations of the technique in question, especially in multiphase systems and in high-precision contexts, such as the monitoring of Small Modular Reactors.

1. INTRODUCTION

Radioisotopes have been widely used in research and technological development around the world, as their measurements are reliable and make it possible to use non-intrusive methods in various dynamic systems ^[1]. As radiotracers, they are widely used in various industrial applications, making it possible to monitor and analyse processes and optimise their performance and efficiency ^[2].

In 2023, Vesvikar and colleagues published a study on Radioactive Particle Tracking (RPT), where the Data Acquisition Frequency (DAF) used was 50 Hz (50 samples per second).



It is worth emphasising that the DAF should not be too high or too low to track particles in real time, as high values above 200 Hz introduce noise into the acquired data and low frequencies can cause dynamic errors in the signal reconstruction. The permissible lower limit of the acquisition frequency depends on the maximum speed of the moving particles and, to calibrate the system, each particle will have to be placed individually in previously known locations [3].

Accurate monitoring of the RPT technique requires the detection of a large number of photons, since the detected value is subject to statistical fluctuations due to the random nature of radioactive decay and the detection process [4]. To track the movement of a fast particle in real time it is necessary to use high sampling frequencies of up to 100 Hz, so it is possible to determine its position every 0.01s. It is important to emphasise that this is possible when the microparticle has sufficient radioactivity and a detection system with a high counting capacity [5].

The relationship between sensitivity, resolution and frequency in RPT data collection for multiphase systems is complex. Kamalanathan (2017) and Upadhyay (2019) have provided additional information regarding the practical implementation of RPT, but have not directly addressed the mathematical relationships between performance parameters and DAF [6]. For this reason, although it is clear that sensitivity, resolution and frequency of data collection are crucial factors for RPT, the relationships between these factors have not yet been established. It is therefore extremely important to try to formulate methods capable of optimising these parameters and thus investigate how these factors are related to the speed of the particle in the medium [7]. The aim is to reduce errors in the reconstruction of the position of the radiotracer particles in order to improve the accuracy and spatial resolution of the technique.

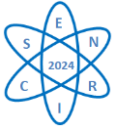
In this context, the aim of this work is to derive simple relationships between the performance interruptions of the RPT technique according to the test conditions and thus provide the permitting functions to solve problems related to detector geometry, particle activity and data acquisition frequency. Future experimental tests will be carried out on a small-scale physical prototype of a Small Modular Reactor (SMR), developed by researchers from the Reactor Engineering group of the Nuclear Energy Department of the Federal University of Pernambuco (DEN/UFPE), in conjunction with the Northeast Regional Nuclear Science Center (CRCN-NE).

2. METHODOLOGY

According to Rasouli et al. (2014), the number of gamma rays detected (event counts) by a detector is influenced by the relative position of the tracer and the material located between the detector and the tracer [8]. This relationship can be approximated by a phenomenological model that links the number of detected events at detector j to the tracer's position x , $C_j(x)$, as expressed in equation 1.

$$C_j(x) = \frac{T \cdot v \cdot A \cdot \Phi_j \cdot \varepsilon_j(x, \mu_j)}{1 + \Gamma \cdot v \cdot A \cdot \Phi_j \cdot \varepsilon_j(x, \mu_j)} \quad (1)$$

Where T is the sampling time interval, v is the number of photons per disintegration, A is the activity of the radioisotope, Φ_j is the peak-to-total ratio of the radioisotope, $\varepsilon_j(x, \mu_j)$ is the total efficiency of detector j , μ_j is the attenuation coefficient of the mean and detector material and Γ is the dead time of detector j . The total efficiency $\varepsilon_j(x, \mu_j)$, which involves the attenuation coefficient, is the key variable dependent on the tracer's location and presents the greatest challenge for evaluation. The product $\Phi_j \cdot \varepsilon_j(x, \mu_j)$ is called the full-energy peak efficiency of the detector for the radioisotope.



Full-energy peak efficiency is defined as the ratio of the number of photons with energy E detected in a photopeak (N_j) to the total number of photons emitted at the same energy (N_E) during a given time interval T (Knoll, 2016). This absolute efficiency, expressed as a dimensionless fraction, is dependent on the specific geometry between the source and the detector, as well as the method used for peak analysis ^[9].

The efficiency can be determined experimentally by mapping the detection efficiencies in off-center positions and as a function of radial distance, and its total peak energy efficiency is described by equation 2 ^[10].

$$\varepsilon_p(x, d + d_0) = \varepsilon_p(0, d + d_0) \cdot \frac{(d+d_0)^2}{d^2+(d+d_0)^2} \quad (2)$$

Where ε_p is point-source efficiency, x is the radial distance, d is the distance from the source center to the detector end-cap, and d_0 is the distance from the detector end-cap to the point detector.

Consider a point radioactive source moving along the X axis in a rarefied medium with a scalar velocity V . In addition, there is a detection system consisting of N identical detectors, with the same point source efficiency ε , and uniformly distributed in a plane perpendicular to the X axis. The detectors are distributed equidistant from the X axis along which the source moves (longitudinal distance d), as shown in Fig. 1.

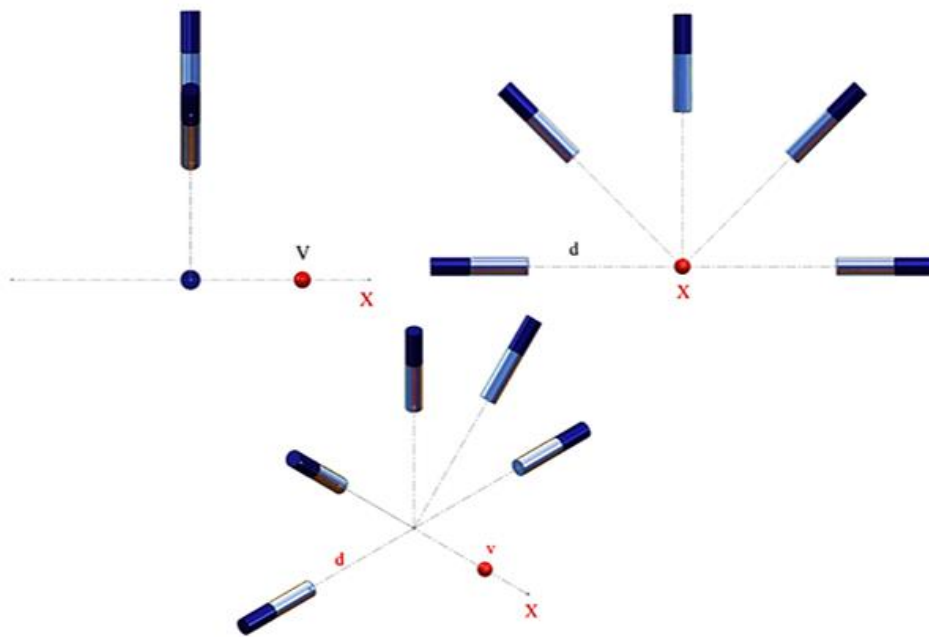
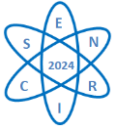


Fig. 1. Representation of the RPT technique detection system.

Let m be the count rate in the detectors and M the value of the net count rate, after subtracting the count rate from the background radiation B and correcting for the detector dead time Γ (equation 3) ^[11].

$$M = \frac{m}{1 + \Gamma \cdot m} - B \quad (3)$$



The spatial resolution R refers to the system's ability to distinguish between two closely spaced positions of a radioactive particle. Here, it is given by equation 4, where σ is the standard uncertainty of the measurements of the counts, C , during the sampling period T , that is, $C=M.T$ and x is the source position varying along the X axis.

$$R = \sigma \frac{dx}{dC} \quad (4)$$

The sensitivity S of the detection system, given by equation 5, measures its ability to respond to changes in the counts C between two closely spaced positions of the radioactive particle.

$$S = \frac{1}{C} \frac{dC}{dx} \quad (5)$$

In a RPT system, let's consider that a sequence of source positions along X is reconstructed so that the uncertainty at each position has the value L . For simplicity's sake, it is assumed that the source emits radiation isotropically and that the attenuation of the radiation from the medium is negligible. As a consequence of the symmetry of the distribution of the detectors around X , where the source moves from position $x-L/2$ to position $x + L/2$, the variation Δm in the counts is the same for the N detectors, as it only depends on the variation in the geometric efficiency ($\Delta \epsilon$) which, for the same reason, is also the same for all of them. The value of Δm is given by equation 6, where ν is the number of photons emitted per disintegration and A is the activity of the radioisotope.

$$\Delta m = \nu . A . \Delta \epsilon \quad (6)$$

The conditions established for calculating Δm ensure that the sensitivity value S , calculated in equation 3, is maximized. Since, by definition, resolution is inversely proportional to S , the corresponding value of R will be as small as possible, as desired.

The usual condition for the time interval T between two consecutive source positions is $T \gg \Gamma$, i.e. the detectors have dead time values lower than T and the photon count C in a detector is given by equation 7.

$$C = mT - BT \quad (7)$$

The variation in counts ΔC in a detector when the source moves from $x-L/2$ to $x+L/2$, considering L is small enough for the variation in B to be negligible, is given by equation 8.

$$\Delta C = T . \Delta m \quad (8)$$

The geometric efficiency of a circular detector of radius r in relation to the point source at position x is given by equation 9, where d is the distance between the center of the detector and the X axis.

$$\epsilon(x, d) = \left(\frac{1}{2}\right) \cdot \left(1 - \frac{d}{\sqrt{d^2 + r^2}}\right) \cdot \frac{d^2}{x^2 + d^2} \quad (9)$$



By combining equations 6 to 9, it is possible to integrate the counts from 0 to ΔC during the displacement of the point source around X, $\Delta x = (x+L/2) - (x-L/2) = L$; Then, the value of ΔC is given by equation 10.

$$\Delta C = v \cdot A \cdot D(d, r) \cdot T \cdot \left(\frac{-2 \cdot x \cdot L}{(x^2 - L/2)^2} \right) \quad (10)$$

Given that $D(d, r)$, in equation 11, is a constant that depends only on the values of d and r .

$$D(d, r) = \left(\frac{1}{2} \right) \cdot d^2 \cdot \left(1 - \frac{d}{\sqrt{d^2 + r^2}} \right) \quad (11)$$

Considering that V indicates the particle velocity and $f=1/T$ is the data acquisition frequency, V/f is the displacement Δx of the source. Now, the derivative on equation 3 can be replaced by the $\text{Lim}(\Delta C/\Delta x)$ when Δx tends to zero, using equation 8, to obtain the relationship between resolution R as a function of x and f , as expressed by the analytical formula in equation 12.

$$R(x, f) = \frac{\sqrt{f} \cdot \{3 \cdot (V/f) \cdot x^2 - (V/f)^2 \cdot x + (V/f)^3\}}{2\sqrt{x^2 + d^2} \cdot \sqrt{v \cdot A \cdot D(d, r)}} \quad (12)$$

The final resolution for N detectors is given by equation 13.

$$\frac{1}{R_N^2} = \sum_{i=1}^N \frac{1}{R_i^2} \quad (13)$$

As the detectors are identical and their centres are always equidistant from the source during the displacement Δx , the individual resolutions are equal to $R(x, f)$. Therefore, the final resolution R_N of the detection system in the experimental setup, as seen in figure 1, is given by equation 14.

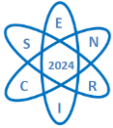
$$R_N(x, f) = \frac{R(x, f)}{\sqrt{N}} = \frac{\sqrt{f} \cdot \{3 \cdot (V/f) \cdot x^2 - (V/f)^2 \cdot x + (V/f)^3\}}{2\sqrt{x^2 + d^2} \cdot \sqrt{N \cdot v \cdot A \cdot D(d, r)}} \quad (14)$$

3. RESULTS

To investigate the variation of resolution as a function of particle position in a detection system, an experimental arrangement was utilized as illustrated in figure 1 and defined in Tab.1, the three minimum values of the R_N function, as described by equation 12, were calculated by varying the frequency.

Tab 1. Experimental Radioparticle Detection Parameters

Radioparticle Activity (Bq)	Distance from detector to X axis (cm)	Detector radius (cm)	Particle speed (cm/s)	Number of detectors (N)	Data acquisition frequency (Hertz)
3,7 x 10 ⁶	30,00	2,54	100,00	4	10 Hz 100 Hz 200 Hz
Minimum R_N point (cm): $x = 0,00$ cm			Minimum values ($\times 10^{-6}$ cm)		6000 19,2 3,39



The graph in Fig.2 shows the relationship between resolution (in centimeters) and position on the X axis (in centimeters) for three different data acquisition frequencies: 10 Hz, 100 Hz and 200 Hz. The curves are identified by three colours: blue, orange and green, respectively.

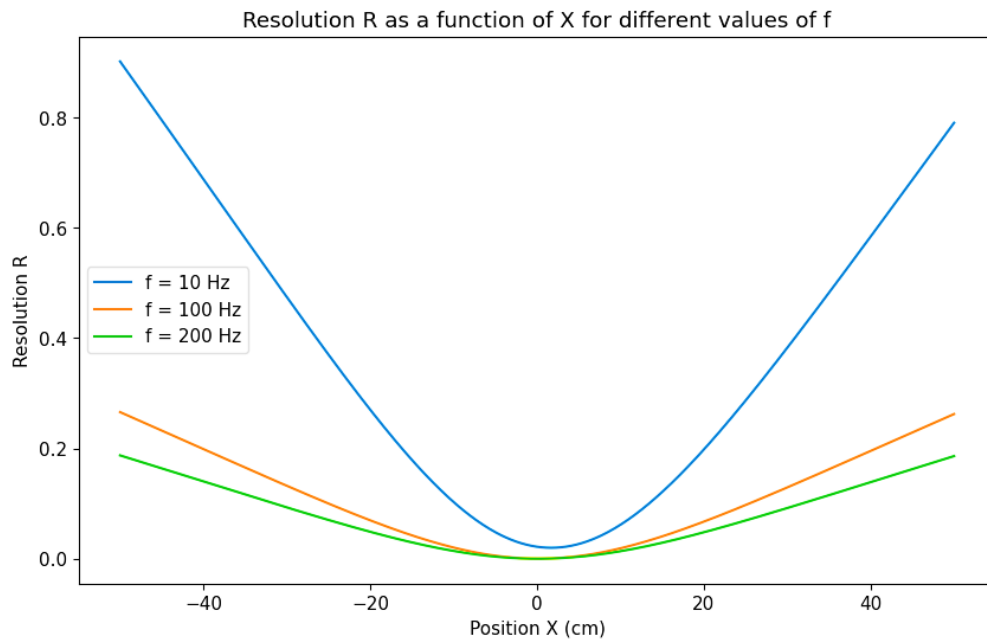
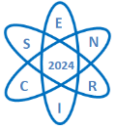


Fig. 2. Resolution (in cm) of the detection system as a function of the position X of the radioparticle, which is moving at a constant speed $v = 100$ cm/s.

Optimizing the resolution of a detection system is of the utmost importance, as it represents the ability to discriminate, which is directly associated with the precision and quality of the specificities made. It is common to observe that the resolution initially improves (decreases) until it reaches a minimum point, after which it deteriorates (increases) as a particle moves away from the ideal position for the detector configuration. This behavior can be attributed to both the geometry of the detection system and the spatial arrangement of the detectors. From an experimental point of view, the X position where the best resolution is obtained suggests the existence of an optimum point where detection is maximally accurate, possibly due to the convergence of the signals coming from the different detectors or the optimization of data acquisition at that specific point.

The results indicate that the frequency of 10 Hz ($f = 10$ Hz) has worse resolution compared to the other frequencies shown. This suggests that a low acquisition frequency is associated with lower particle detection accuracy. The orange curve ($f = 100$ Hz) shows an interesting resolution, presenting a specific improvement over the blue curve, corroborating the possibility that increasing the frequency is directly related to an improvement in specificity accuracy. The green curve ($f = 200$ Hz) reveals the best resolution across the entire range of positions, showing that a higher data acquisition rate allows for more accurate detection. Furthermore, the improvement in resolution with increasing frequency can be explained by the system's greater ability to capture rapid variations in the signal, allowing for a more detailed analysis of particle behavior. This positive relationship between the acquisition frequency and the accuracy of the results highlights the importance of updating the experimental conditions in order to guarantee the effectiveness of the detection. Therefore, choosing an appropriate



acquisition frequency not only improves the quality of the data obtained, but also enhances the reliability of the connections extracted from this data, contributing to a deeper understanding of the system.

The graph in Fig. 3 illustrates the variation in resolution as a function of frequency for three positions: the minimum RN point and two positions symmetrical to it, located at $x = -20.0$ cm and $X = 20.0$ cm, for comparison purposes. As expected, the curves corresponding to the symmetrical positions are congruent and there is a convergence in resolution with increasing frequency.

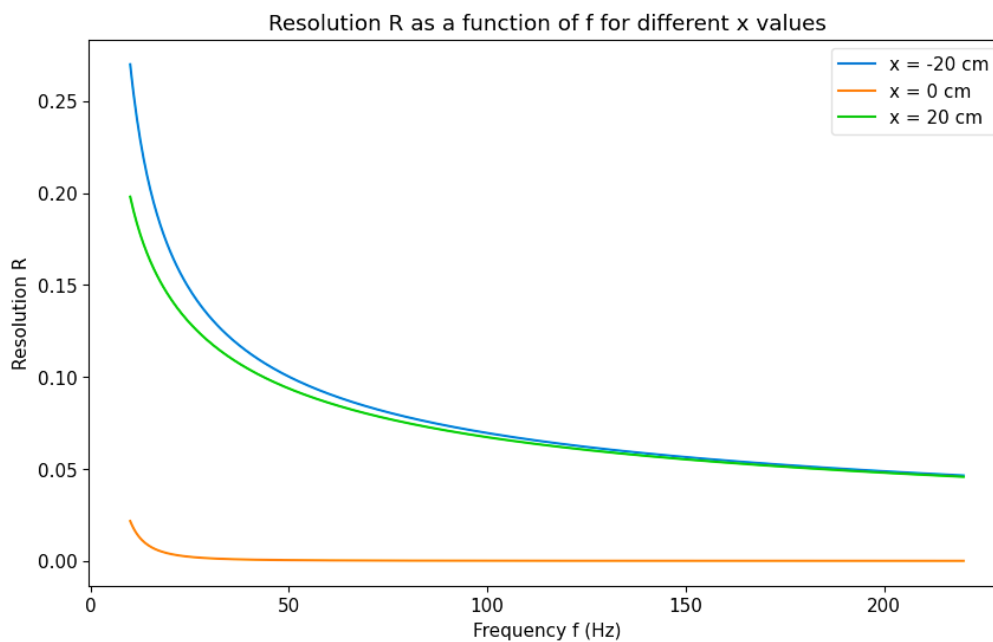


Fig. 3. Resolution of the detection system as a function of frequency for a radioparticle with a scalar velocity of 100 cm/s.

The results show that the variation in resolution as a function of acquisition frequency may indicate the influence of the data collection rate on the system's accuracy. In general, by increasing the acquisition frequency, an improvement in resolution is expected up to a certain point, as data collection intensifies as the displacements approach the instantaneous positions of the particles. However, beyond a specific limit, increasing the frequency may not result in a significant improvement in resolution due to system limitations, such as noise or processing capacity. Therefore, it can be inferred that the ideal data acquisition frequency will be the one that provides the best resolution for the system, without compromising efficiency or introducing unwanted noise. The point at which the resolution stabilizes or begins to increase again (the later not seen in this theoretical approach) may reflect the system's capacity or the introduction of noise.

4. CONCLUSION

The results corroborate that the mathematical model expressed in equation 14 makes it possible to analyze the variation in resolution as a function of position and frequency for a given particle velocity. Furthermore, the equation can be used as a tool to optimize both the experimental setup and the minimum required activity of the radioactive particle, in order to achieve the best performance in the tests, in favor of the radiological protection of personnel



involved in experiments. Thus, it is important to note that the acquisition frequency must be chosen carefully in order to maximize resolution without introducing additional limitations. The study reveals that although increasing the frequency improves resolution, there is a point at which the additional benefits become insignificant or even counterproductive. Therefore, identifying the optimum frequency is crucial for the effective performance of the detection system.

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BIBLIOGRAPHICAL REFERENCES

- [1] W. S. Vieira et al., An alternative method for tracking a radioactive particle inside a fluid, *Applied Radiation and Isotopes*, Vol. 85, pp. 139-146 (2014).
- [2] A. R. Rammohan et al., Characterization of single phase flows in stirred tanks via computer automated radioactive particle tracking (CARPT), *Chemical Engineering Research and Design*, Vol. 79, pp. 831-844 (2001).
- [3] M. S. Vesvikar et al., Development, validation and implementation of multiple radioactive particle tracking technique, *Nuclear Engineering and Technology*, Vol. 55, pp. 4213-4227 (2023).
- [4] S. Roy, Optimal design of radioactive particle tracking experiments for flow mapping in opaque multiphase reactors, *Applied Radiation and Isotopes*, Vol. 56, pp. 485-503 (2002).
- [5] R. K. Upadhyay and S. Roy, Resolution, Sensitivity and Accuracy of the Radioactive Particle Tracking Technique, 8th World Congress of Chemical Engineering WCCE8 2009, p. n.p, Montreal, 23 august (2009).
- [6] P. Kamalanathan et al., Effect of dynamic bias on accuracy of radioactive particle tracking (RPT) technique at different data acquisition frequencies, *Applied Radiation and Isotopes*, Vol. 128, pp. 13-21 (2017).
- [7] R. K. Upadhyay et al., Experimental validation of design and performance parameters of radioactive particle tracking (RPT) experimentation, *Applied Radiation and Isotopes*, Vol. 153, p. n.p, (2019).
- [8] M. Rasouli et al., A Multiple radioactive particle tracking technique to investigate particulate flows, *Particle Technology and Fluidization*, Vol. 61, pp. 384-394 (2014).
- [9] G. F. Knoll, *Radiation Detection and Measurement*, 4^o ed., Nova Jersey, Ed. Wiley (2016).
- [10] J. C. Aguiar, An analytical calculation of the peak efficiency for cylindrical sources perpendicular to the detector axis in gamma-ray spectrometry, *Applied Radiation and Isotopes*, Vol. 66, pp. 1123-1127 (2007).
- [11] S. Usman and A. Patil, Radiation detector deadtime and pile up: a review of the status of science, *Nuclear Engineering and Technology*, Vol. xx, p. n.p, (2018).