OPTIMIZATION OF NUCLEAR FUEL BURNUP THROUGH THE LINEAR REACTIVITY MODEL

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ABSTRACT

This work aims to describe the reactivity of a nuclear fuel rod as a function of burning. For this, the Linear Reactivity Model (LRM) is used. The fundamental hypothesis is based on the linear dependence between reactivity and axial burnup. As the second one is dependent on the axial power distribution profile, new distributions are studied that satisfy the boundary requirements in the diffusion model. The results obtained by the Linear Reactivity Model, for the maximum Burnup, are quite accurate when compared with the the literature. However, it was advantageous for pointing out a smaller region of differentiated enrichment at the ends of the rod, extending the application of the Linear Reactivity Model to small modular reactors (SMR).

1. INTRODUCTON

This work aims to describe the reactivity of a nuclear fuel rod as a function of burning. For this, the Linear Reactivity Model (LRM) is used. The fundamental hypothesis is based on the linear dependence between reactivity and axial burning. As the second one is dependent on the axial power distribution profile, new distributions that satisfy the boundary requirements in the diffusion model are studied [1]–[4].
The starting point of this work focuses on the presentation of the Linear Model of Reactivity described by Driscoll, et al 1981 [5], as well as in measuring the results of their work.
Subsequently, a new mathematical structure will be proposed in order to update the model by [5], and then compare their results with previous work.
Finally, other results will be presented related to the profile of maximum fuel burning, and how the enrichment should be applied to the rod so that the highest burning efficiency can be achieved.

2. METHODOLOGY

For a better understanding of the work, it is necessary to grasp concepts related to burnup and enrichment of fuel rods.

2.1. Theorical Basis

For PWR-type reactors, an approximation can be made that consists in assuming that the reactivity ρ is a function of the burn state (Q). In this way, it becomes possible to observe that ρ is a function with a single value determined by Burnup, without necessarily depending on the temporal accumulation of Burnup [6].

Although some preliminary assumptions are necessary, the main consideration to be made is to assume that the reactivity ρ can be described in the form of a burn summation polynomial, such that expressed in Equation 2.1:

$$\rho = \rho_0 + \sum_{i=1}^{i=n} A_i Q^i \tag{2.1}$$

Where ρ_0 is the initial reactivity, A_i a coefficient that must be determined and Q^i the burning. This mathematical relationship is based on Light Water Reactors (LWR), whose observations made in the 1950s urged the existence of a linear relationship between reactivity and burning [5].

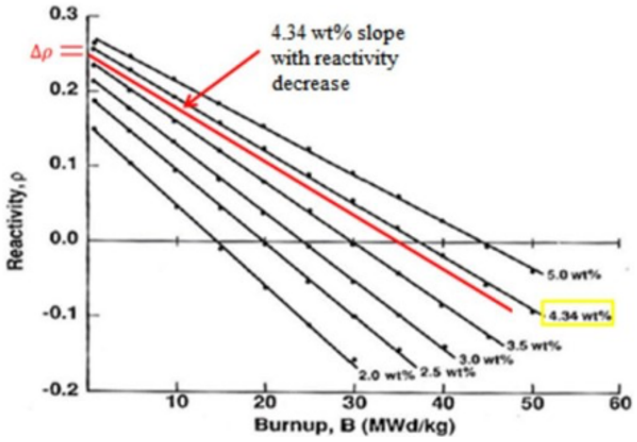

Figure 1. Relation between Burn (B) and Reactivity

According to Figure 1, it is noticed that the reactivity tends to decrease with the increase of the firing, verifying that the linearity relation has a negative angular coefficient. Another point refers to the enrichment (wt%), where its value is directly associated with the linear coefficient.
In the Linear Reactivity Model (LRM) terms of second order or more are disregarded, thus making the function ρ a linear function of Burnup, thus obtaining:

$$\rho = \rho_0 + A_1 Q \tag{2.2}$$

Where in Equation 2.3 the term A_1 is considered as the rate of change of reactivity in relation to burning, in which its units must be interpreted as the inverse of the burning unit. That is, A_1 has the units of MTU/MWd.
As shown by theory, burning is a function that depends on the point z, on the axial axis on the fuel rod. Thus, it is logical to assume that reactivity is also a spatially dependent function. So, Equation 2.3 becomes:

$$\rho\,(z) = \rho_0\,(z) - A_1 Q\,(z) \tag{2.3}$$

The main problem with Equation 3 resides in the fact that the function $\rho(z)$ must be interpreted only as an auxiliary function. Because, from an experimental point of view, it is not possible to measure the reactivity locally, as well as the multiplication factor k. This occurs due to the fact that ρ and k are global functions [6].
Moving forward, the analysis focuses on a way to define $Q(z)$. For this, consider a fuel rod of length 2b centered at the origin O of the z-axis, as shown in the figure below:

Figure 2. Idealization of a fuel rod of length 2b

Assuming that the burn $Q(z)$ is proportional to the reactor flow and/or power, the local burn can be written as being proportional to a function $f(z)$, such that $Q(z) \propto f(z)$, as follows [6]:

$$Q\left(z\right) = \alpha f\left(z\right) \tag{2.4}$$

$f(z)$ is the axial weighting function and represents the power per unit length. The normalization condition imposes that [6]:

$$\int_{-b}^{b} f\left(z\right)\, dz \;= 1 \tag{2.5}$$

Once this is done, any function average can be computed by weighting the function of interest. Therefore, the local burn $Q(z)$ can be determined in terms of the average burn $\overline{Q}$, obtaining then:

$$Q\left(z\right) = 2b\overline{Q} f\left(z\right) \tag{2.6}$$

Through these definitions, one can then multiply Equation 2.3 on both sides by $f(z)$ and integrate over the entire length of the rod (2b), to obtain:

$$\overline{Q} = \frac{\rho_L - \overline{\rho_0}}{2bA_1 \int_{-b}^{b} \left[f(z)\right]^2 dz} \tag{2.7}$$

where ρ_L is defined by Driscoll et al., 1981 [5] as the so-called "Escape Reactivity", being expressed using the neutron current J(z) in the following form [6]:

$$\rho_L \;=\; \int_{-b}^{b} J\left(z\right) f\left(z\right)\, dz \tag{2.8}$$

2.2. The Weight Functions $f(z)$

In the present work, the resource is to propose an adequate form for the function $f\left(z\right)$ that allows to reduce the temperature peak, and as a consequence, to optimize firing, making the necessary

changes to the rod. In their work, assumes a fuel rod of length 2(a+b) with two different enrichment zones, as shown in Figure 3 [5].

Figure 3. Idealization of a fuel rod with different enrichment zones

From here two considerations can be made. The first consideration refers to the fact that when we make axial changes in the rod, different axial profiles of firing will result [6]. The second point focuses on ρ_L. The Linear Reactivity Model has the assumption that radial leakage is negligible in relation to axial leakage, therefore, in reactors where this assumption is not verified, the LRM cannot be applied [6].
Thus, by hypothesis it is supposed that $f(z)$ presents a constant behavior between –b to b, and in the region of size a, $f(z)$ is expressed as a trigonometric function. Obtaining then [6]:

$$\begin{cases} f(z) = C, & 0 \leq z \leq b. \\ f(z) = C \sin\left(\frac{\pi}{2a}[a+b-z]\right), & b \leq z \leq (a+b). \end{cases} \tag{2.9}$$

Where the constant C can be determined using the definition of normalization given by Equation 2.5. It is worth noting that the trigonometric form for f(z) can be changed to a cosine as long as the appropriate adjustments are made to the angular parameters. Therefore, the axial power distribution can be seen in Figure 4.

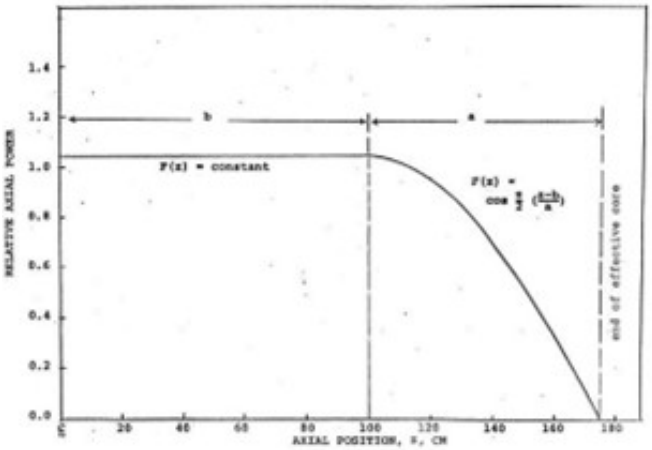

Figure 4. Axial distribution of power [5]

In the present work, the authors propose an alternative form for $f(z)$ that does not require the separation of the fuel rod by different enrichment zones as done by. Therefore, the function f(z)

is defined over the entire length 2b of the rod as shown in Figure 2 in the following mathematical structure:

$$f_n(z) = C\sqrt{1 - \left(\frac{z}{b + \delta}\right)^n} \tag{2.10}$$

Where the index n is any real number, b is the value that corresponds to half the length of the rod, C is the normalization constant and δ is a characteristic extrapolated distance. As an example, Figure 5 contains the behavior for $f_8(z)$.

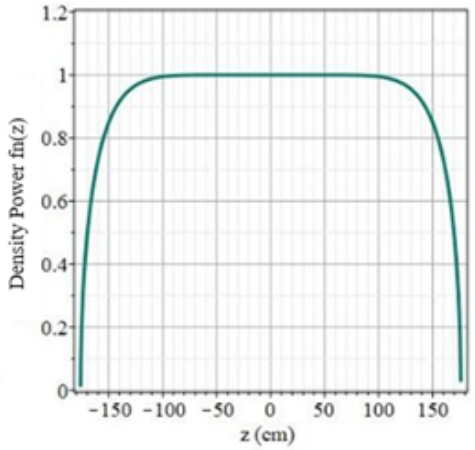

Figure 5. Axial power distribution for $f(z)$.

From this point on, having defined the $f(z)$ and $f_n(z)$ functions, the work focuses on determining the maximum values for Equation 2.7.
Simulated Fuel Rod For the execution of this work, computational simulations were carried out using Maple software and Python codes for a reactor with the following conditions.

- Migration Length M = 7.5 cm.
- Reactor length 2L=350 cm.
- Half the length of the rod b=L- a.
- Coefficient of variation of reactivity $A_1 = 0,91 \times 10^{-5}$
- Initial reactivity $\rho_0 = 0.2$

3. RESULTS

3.1. Results for Medium Burnup $\overline{Q}$

The Figure 6 represents the average burn $\overline{Q}$ as a function of the differentiated enrichment region of size a.

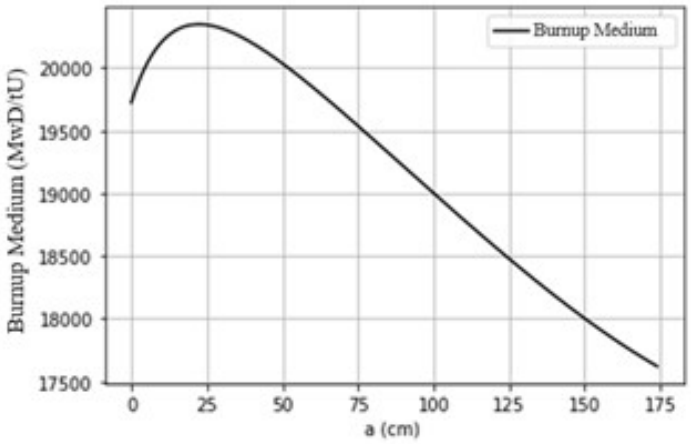

Figure 6. Average burn ($\overline{Q}$) according to the size of region a

In Figure 6 it is possible to extract some results. The first refers to the fact that when a ≈ 0, $\overline{Q}$ is relatively low and corresponds to the distribution $f(z) = C$. The second resides in the fact that when $a \approx b$ the distribution $f(z) \propto \cos\left(\frac{\pi}{2b}z\right)$ and the average burn is drastically reduced.
Therefore, it is possible to perceive the existence of an intermediate point for a, where $\overline{Q}$ will present a maximum. In this specific case, where M=7.5 cm and the rod has an active length of 350 cm, the maximum average burn is $\overline{Q} \approx 20330 \frac{MWd}{tU}$ for a value of an approximately equal to 23 cm, in perfect agreement [2].
Analyzing now the behavior of $\overline{Q}$ for $f_n(z)$, we must first highlight the fact that there will be a value of $\overline{Q}$ for each value of n. In order to simplify the analyses, we chose to work only with natural values for n. Thus, Figure 7 shows the power profiles for 3 different types of n.

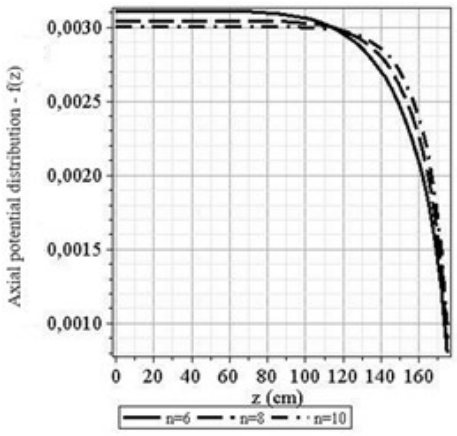

Figure 7. AveraPower distribution $f_n(z)$ for n = 6,8 and 10 .

Figure 8 plots the value of $\overline{Q}$ for some values of n.

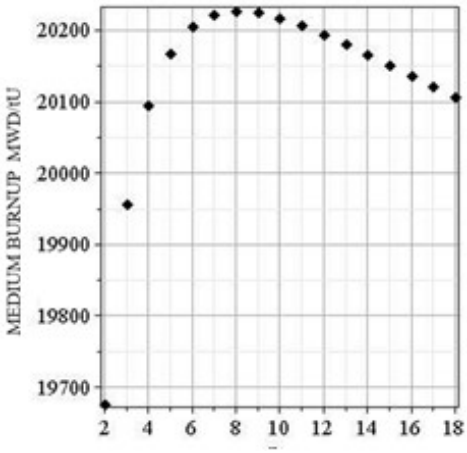

Figure 8. Maximum values for $\overline{Q}$ when b=175 cm .

It can then be seen from Figure 8 that the profile for $f_n(z)$, which presents the highest value for $\overline{Q}$, is when n=8. Therefore, according to Figure 8, it can be stated quite precisely that the function $f_8(z)$ provides the highest average burn value. Once the distribution function has been defined, the calculations of other parameters of interest are carried out. The Table 1 below contains the values of $\overline{Q}$ for some even values of n.

n	Q (MWd/tU)
2	19.653
4	20.091
6	20.216
8	20.250
10	20.241
12	20.233
16	20.210

Table 1. Values of Q to n

3.2. Results For The Enrichment Distribution

It is possible to use the $f_n(z)$, function to define how the enrichment profile will be along the rod, through relation :

$$-D\frac{\nabla^2 f_n(z)}{f_n(z)} \cong A\epsilon(z) + B \tag{3.1}$$

where D is the diffusion coefficient, A and B are constants determined by the boundary conditions and $\epsilon(z)$ is the axial distribution of enrichment.
It is important to assume the boundary conditions that will be used to determine the constants A and B as well as to plot the behavior of the distribution of the enrichment $\epsilon(z)$.

The first condition is that the enrichment of $U235$ in the central part of the rod, $\epsilon\,(0)$ is 2.9wt%. The choice of this condition lies in the fact that it is the same [6]. The second condition allows the maximum enrichment used at the tips of the rods $\epsilon(b)$ to be freely chosen. Thus, in this work, enrichments of 4wt%, 5wt% and 6wt% were used.
As per the results of the previous section, the power density that exhibited the best average burn was$f_8\,(z)$. Therefore, the parameters that will be calculated ahead will all be determined based on the best $\overline{Q}$. Thus, rewriting 2.10 to $f_8\,(z)$ and making D = 1, we have:

$$\epsilon\,(z) = \Delta\epsilon \frac{f_8\,(b)}{\nabla^2 f_8\,(b)} \frac{\nabla^2 f_8\,(z)}{f_8\,(z)} + \epsilon\,(0) \tag{3.2}$$

Where $\Delta\epsilon \equiv \epsilon\,(b) - \epsilon\,(0)$.
Finally, it should be noted that, although Equation 12 is for n=8, its validity is for any n. Then, using all the reasoning exposed so far, it is possible to plot the z-enrichment distribution along the rod. Using the same data from Driscoll et al. 1981[1] for a typical PWR, if we have b= 175 cm and $\delta = 1cm$, we will have as it is shown in the Figure 9:

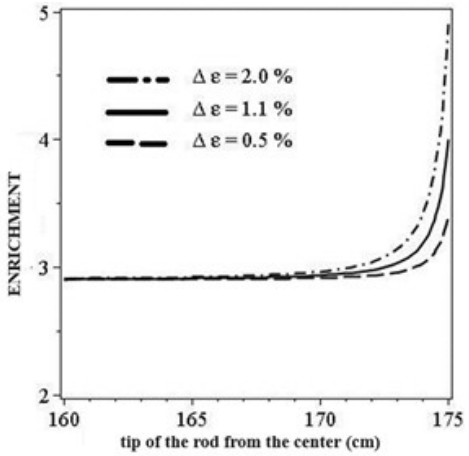

Figure 9. Enrichmente distribuition.

3.3. Discussion of Results

Figure 9 shows how the axial enrichment distribution of the rod changes for different enrichment variants (ie different $\Delta\epsilon$), calculated by the axial enrichment equation. It is interesting to note that, at first, it is easy to be misled into thinking that any $\Delta\epsilon$ can this difference in enrichment is directly linked to the ratio between $\nabla^2 f_n$ and f_n at the end of the stick.
Figure 10 shows how leakage reactivity varies axially for the Driscoll and the present work. It is obtained by integrating the product of the neutron escape fraction by the power distribution $f(z)$ for each model, according to Equation 8. In the model by Driscoll used [5].

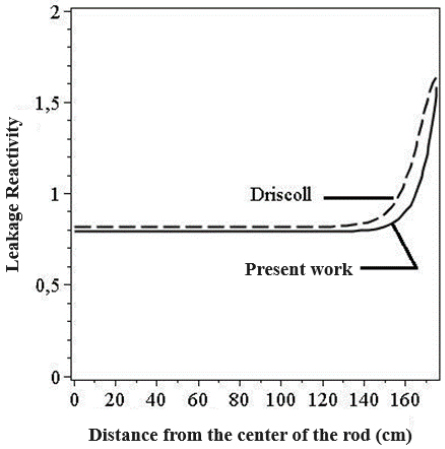

Figure 10. Axial distribution of leakage reactivity ρ_L.

The discretized weighting function. In the present model, the continuous function $f_8(z)$ was used.

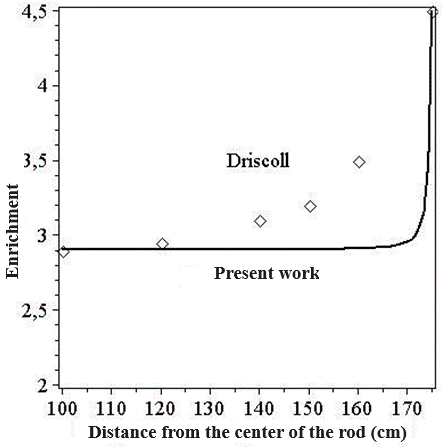

Figure 11. Comparative curve of the axial distribution of enrichment.

Figure 11 shows the axial distribution of enrichment in the present study when compared with the numerical result of the reference article by Driscoll . It is possible to notice that the length, with higher enrichment, is greater in the reference model, extending to at least more than 30 cm, while the present model was restricted to a few centimeters. It should be noted that the second is consistent with the diffusion model.
Figure 12 shows how neutron leakage (related to the current J) is accentuated at the ends of the rod, while it is zero along most of the length. This shows that the definition used for J is in perfect agreement with Fick's Law.

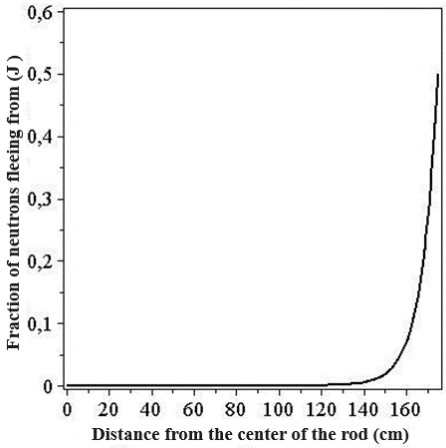

Figure 12. Fraction of neutrons fleeing from J.

Finally, the results of applying the $f_n(z)$ distribution for an SMR-type reactor are presented, where b = 50 cm and e $\delta = 1$, as in Figure 13.

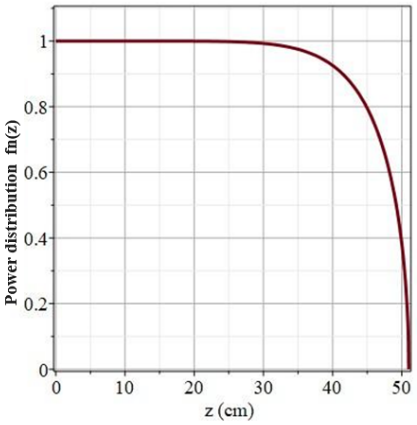

Figure 13. Power distribution for an SMR.

And then the enrichment distribution as follows: For $\Delta\epsilon = 1,1\%$:

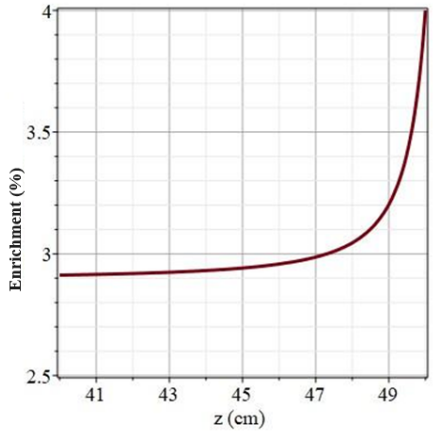

Figure 14. Distribution of enrichment $\epsilon\,(z)$ for an SMR

Figure 14: Distribution of enrichment $\epsilon\,(z)$ for an SMR (PONTES, 2022).
One of the most important results found can be seen in Figure 14. This result refers to the application of the Linear Reactivity Model for small reactors (SMR). If we use the f(z) model segmented by regions [6], the results indicate that to reach the maximum $\overline{Q}$ it is necessary that the differentiated enrichment region be implemented in about 50% of the fuel rod . However, the analysis of the model for $f_n\,(z)$ allowed to verify that it is possible to reach a maximized $\overline{Q}$ profile using a region of approximately 6% of the length of the rod . Therefore, the application of the $f_n\,(z)$ model is more effective for the treatment of an SMR, compared to the $f\,(z)$ divided by regions model by Driscoll **.**

4. CONCLUSION

The first conclusion to be mentioned in this work refers to the accuracy that the analytical model has for solving problems treated numerically in the vast majority of cases. And, as the acquired results are very close to the results of Driscoll et al. (1981), who is the main reference on which this work is based. Another point that should be highlighted is the possibility of finding a simple mathematical method to work in the $\overline{Q}$ optimization process. Compared to the model made by Driscoll et al. (1981), the proposition of a function $f_n\,(z)$ allows all mathematical analysis to be performed without the need to separate the fuel rod into regions.
The last conclusion of this work is the verification of the possibility of obtaining the same results for maximizing the $\overline{Q}$ of the fuel, as performed in the work of Driscoll et al. (1981). Through the function segmented by n, ie $f_n\,(z)$ it was possible to find maximum $\overline{Q}$ values similar to those found by Driscoll et al. (1981), using segmented $f\,(z)$ found differences of approximately 3.4% for the maximum $\overline{Q}$ value.
Finally, it should be noted that a fuel rod cannot be manufactured with a continuous distribution of enrichment. Finite length inserts would have to be used on the tips. As a result, the central power distribution would lose its constant character a little, moving away from the analytical ideal.

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References

[1] R. R. Ratnakar and V. Balakotaiah, "Multi-scale reduced-order models of electrified wire reactors for carrying-out endothermic reactions", *Chemical Engineering Science*, vol. 295, p. 120 130, 2024, issn: 0009-2509. doi: https://doi.org/10.1016/j.ces.2024.120130.

[2] A. Amatullah, S. Permana, D. Irwanto, and A. H. Aimon, "Comparative analysis on small modular reactor (smr) with uranium and thorium fuel cycle", *Nuclear Engineering and Design*, vol. 418, p. 112 934, 2024, issn: 0029-5493. doi: https://doi.org/10.1016/j.nucengdes.2024.112934.

[3] V. Castro, R. Miró, C. Pereira, and G. Verdú, "A burnup credit methodology for pwr spent fuel storage pools: Application to a standard pwr nuclear power plant", *Nuclear Engineering and Design*, vol. 419, p. 112 948, 2024, issn: 0029-5493. doi: https://doi.org/10.1016/j.nucengdes.2024.112948.

[4] K. Park, S. Cha, J. Sung, *et al.*, "Estimating spent fuel burnup with neutron measurements: A practical rule of thumb equation", *Nuclear Engineering and Technology*, 2024, issn: 1738-5733. doi: https://doi.org/10.1016/j.net.2024.05.027.

[5] M. J. Driscoll, T. J. Downar, and E. E. Pilat, "The linear reactivity model for nuclear fuel management", *(No Title)*, 1990.

[6] M. Melik, "Optimization of the axial power shape in pressurized water reactors", Ph.D. dissertation, Massachusetts Institute of Technology, 1981.